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Are four neutrino models ruled out?

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We show explicitly that four neutrino models of the 2+2 variety still provide an acceptable global fit to the solar, atmospheric and LSND neutrino data. The goodness of fit, defined in the usual way, is found to be 0.26 for the simplest such model. That is, we find that there is a 26% probability of obtaining a worse global fit to the neutrino data We also make some specific comments on the paper, "Ruling out four-neutrino oscillation interpretations of the LSND anomaly" [hep-ph/0207157], and explain why they reached drastically different conclusions.

One area of excitement in physics is coming from the direction of various neutrino experiments. There is now a range of evidence, from atmospheric, solar and terrestrial experiments strongly supporting the idea that neutrinos have mass and change flavour via oscillations. At the present time, the precise oscillation pattern remains uncertain and is the subject of much speculation.

Recently, it was argued [1] that the totality of the neutrino data now suggests an essentially unique picture:

$$\begin{aligned}
&\nu_e \to \nu_\tau \ \text{ large angle oscillations explains the solar neutrino problem} \\
&\nu_\mu \to \nu_s \ \text{ large angle oscillations explains the atmospheric neutrino anomaly} \\
&\bar{\nu}_e \to \bar{\nu}_\mu \ \text{ small angle oscillations explain the LSND data}
\end{aligned} \tag{1}$$

where ν_s is a hypothetical (effectively) sterile neutrino. Although this scheme is not very popular it at least has the virtue that it will be tested in the near future: MiniBooNE will test the oscillation explanation of the LSND anomaly, while the forthcoming long baseline experiments will discriminate between the $\nu_\mu \to \nu_s$ and $\nu_\mu \to \nu_\tau$ possibilities for resolving the atmospheric neutrino anomaly.

Anyway, the aim of this brief note is to explicitly estimate the overall goodness of fit (g.o.f) of the above scheme [Eq.(1)]. We find that the g.o.f is 0.26 which is reasonable, but is perhaps controversial because this value is more than 5 orders of magnitude larger than the value found in Ref. [2]. As we will discuss, the origin of this discrepancy is quite easy to explain.

The oscillation scheme [Eq.(1)] is essentially unique in the sense that it is the simplest scheme involving only two-flavour oscillations explaining the totality of the data and also specific features such as SNO's neutral current/charge current solar flux measurement [3]. Of course, other, but more complicated schemes involving multi-flavour oscillations, are possible because they can also provide an acceptable fit to the data for a range of parameters. For example, one can have an additional parameter, $\sin^2\omega$, where $\sin^2\omega = 0$ corresponds to the scheme, Eq.(1), $\sin^2\omega = 1$ is similar to Eq.(1) with ν_τ interchanged with ν_s and intermediate values of $\sin^2\omega$ corresponds to mixed active+sterile oscillations [4]. Such schemes are called 2+2 models because they feature two pairs of almost degenerate states separated by the LSND mass gap. While the scheme of Eq.(1) could be viewed as a particular 2+2 scheme with $\sin^2\omega = 0$, it could alternatively be viewed as motivating the following hypothesis [1]: The fundamental theory of neutrino mixing, whatever it is, features (i) large (or even maximal) $\nu_\mu \to \nu_s$ mixing, (ii) small-angle active-active mixing except for the $\nu_e \to \nu_\tau$ channel which is large.

Certainly the atmospheric fit of $\nu_\mu \to \nu_s$ oscillations is not perfect, having a goodness of fit of 0.055 [5]. Essentially there seems to be two main possibilities here. First, it might be that the atmospheric neutrino anomaly is due to (predominantly) $\nu_\mu \to \nu_\tau$ oscillations and the solar neutrino anomaly is also due to (predominantly) active-active oscillations in a 3-flavour scheme called bi-maximal mixing [7]. In this case, it suggests that the LSND anomaly is not caused by neutrino oscillations at all. The other possibility is the one identified above in Eq.(1). While the fit to the atmospheric data is not perfect in the scheme [Eq.(1)] it does accomodate the LSND data and also has the theoretical virtue of explaining the data using only two-flavour oscillations. Both possibilities are possible [as we will explicitly show in the case of the scheme of Eq.(1)] and only future data can compellingly distinguish between the scenarios.

The overall g.o.f for the scheme, Eq.(1), can easily be evaluated. Things are especially simple, since Eq.(1) involves only two-flavour oscillations. Basically there are three contributions to the χ^2:

$$\chi^2(total) = \chi^2(atm) + \chi^2(solar) + \chi^2(LSND) \tag{2}$$

To avoid controversy, we use the super-Kamiokande result for the atmospheric $\nu_\mu \to \nu_s$ hypothesis [5]. For solar neutrinos, we use the value for $\chi^2(min)$ obtained by Ref. [8] applicable for $\nu_e \to \nu_\tau$ oscillations (this is a fit to both super-Kamiokande and the latest SNO solar neutrino results). Finally, for the LSND experiment we estimate a χ^2 minimum from Figure.16 (taking the bins with E in the range $20 < E/MeV < 52$) and Figure.24 (taking the bins with L/E in the range $0.5 \lesssim L/E \lesssim 1.2$) of Ref. [9]*. The result of these extractions is:

$$\begin{aligned}
\chi^2(atm) &\simeq 222 \ for \ N_{atm} = 190 \ d.o.f. \\
\chi^2(solar) &\simeq 65 \ for \ N_{solar} = 78 \ d.o.f. \\
\chi^2(LSND) &\approx 4 \ for \ N_{LSND} = 8 \ d.o.f.
\end{aligned} \tag{3}$$

This gives a total $\chi^2(total) \equiv \chi^2(atm) + \chi^2(solar) + \chi^2(LSND) = 291$ for $N_{total} \equiv N_{atm} + N_{solar} + N_{LSND} = 276$ d.o.f. This corresponds to a g.o.f of 0.26. That is, there is a 26% probability of obtaining a worse global fit to the neutrino data used. This explicitly demonstrates that the scheme Eq.(1) currently provides a reasonable global fit to the neutrino data.

Of course, it may also be legitimate to worry about whether such a scheme can fit certain subsets of the data. For example, super-Kamiokande identified 3 subsets of data which might potentially discriminate between the $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_s$ solutions to the atmospheric neutrino anomaly [6]. However, this discrimination, was not particularly stringent (super-Kamiokande found that the $\nu_\mu \to \nu_s$ possibility had a g.o.f of about 0.01), and an alternative analysis [10] of the same data gave a much better fit (a g.o.f of about 0.1). Clearly, this discrimination is crucial and is still one of the main issues that needs to be resolved.

As a final remark, let us also consider the recent analysis of 2+2 schemes by Maltoni, Schwetz, Tortola and Valle (MSTV) [2]. In the MSTV paper, it was found that the g.o.f of 2+2 models to the totality of neutrino data (solar, atmospheric and LSND) was just 10^{-6}, a quite remarkable result given our estimate of 0.26 (above) for a particular 2+2 model! It turns out that it is very easy to identify the origin of the discrepancy. Our result above was based on standard g.o.f obtained using standard χ^2 statistics in the usual way, while the result of Ref. [2] was based on a new type of statistics (which we will denote as MSTV statistics). In MSTV statistics, the value of χ^2 (min) is *not* compared with

*These particular bins are chosen because they have more than 5 events, so that the χ^2 statistic should follow the χ^2 probability density function. Of course, the details of how we analyse the LSND data does not strongly affect our conclusion.

the number of d.o.f to obtain a g.o.f. Instead, the value of χ^2 (min) is compared with another $\chi^2 - like$ quantity, obtained by fitting different data sets with different values of the parameter $\sin^2 \omega$ [the parameter describing the particular mixture of active+sterile involved in the solar and atmospheric oscillations] although, physically it can only take one value within the model. Following a certain prescription, MSTV then obtains a quantity which they call g.o.f. Clearly, the MSTV g.o.f is a sort of relative thing obtained by comparing the standard $\chi^2(min)$ with some unphysical $\chi^2 - like$ function. Thus, the g.o.f calculated by MSTV is nothing to do with the standard value, but some other quantity. [This is obvious since as we have seen, it is different to the standard g.o.f by more than 5 orders of magnitude!]. Really, to be meaningful MSTV would have to show that their g.o.f has some *absolute* statistical significance *and* why the standard g.o.f breaks down so badly for four neutrino models.

In summary, we have explicitly demonstrated that at least some four neutrino models still provide a reasonable description to the totality of the neutrino data. Clearly, more experimental input is necessary to distinguish between the various oscillation schemes still allowed by the data before definite conclusions can be drawn. In particular, a clear discrimination between the $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_s$ atmospheric oscillation hypothesis and a confirmation or otherwise of the LSND result by mini-Boone are required.

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